Benha University
Faculty Of Engineering at Shoubra

ECE 411
Antennas & Wave propagations
(2016/2017)

Lecture (6)
Array of Point Sources

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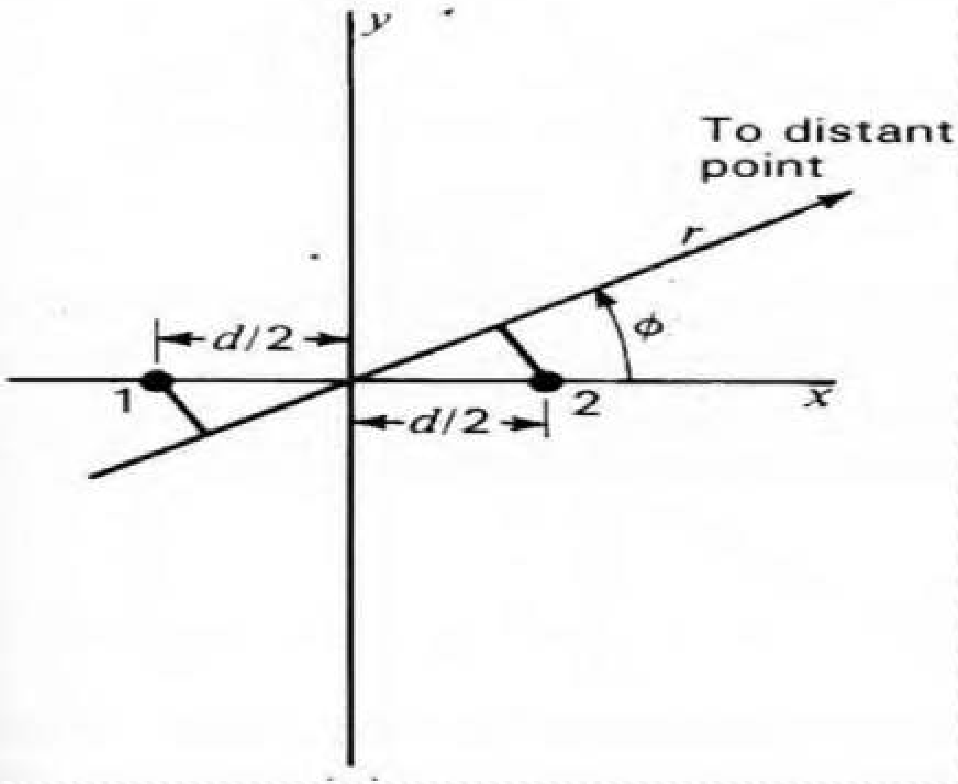
Agenda

- 1 –Remember (array of 2 point sources)
- 2 – Array of N – Isotropic sources
    - End-Fire Array
    - Broad Side Array
    - Examples
- 3 - Array of N (non-isotropic sources)
- 4 - Array of N (non-isotropic sources) (lie on Z)

1- Remember (array of 2 point sources)

Remember (array of 2 point sources)

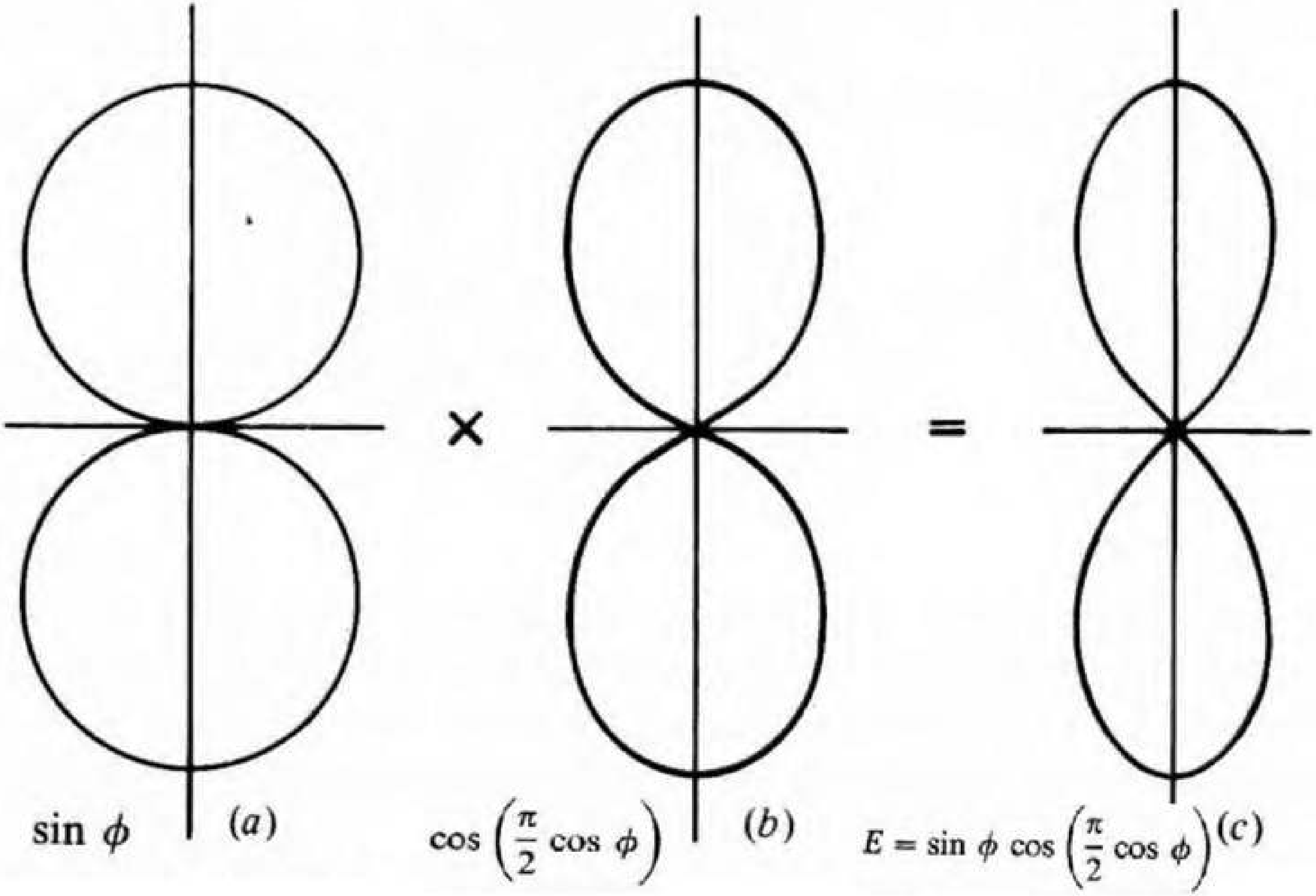


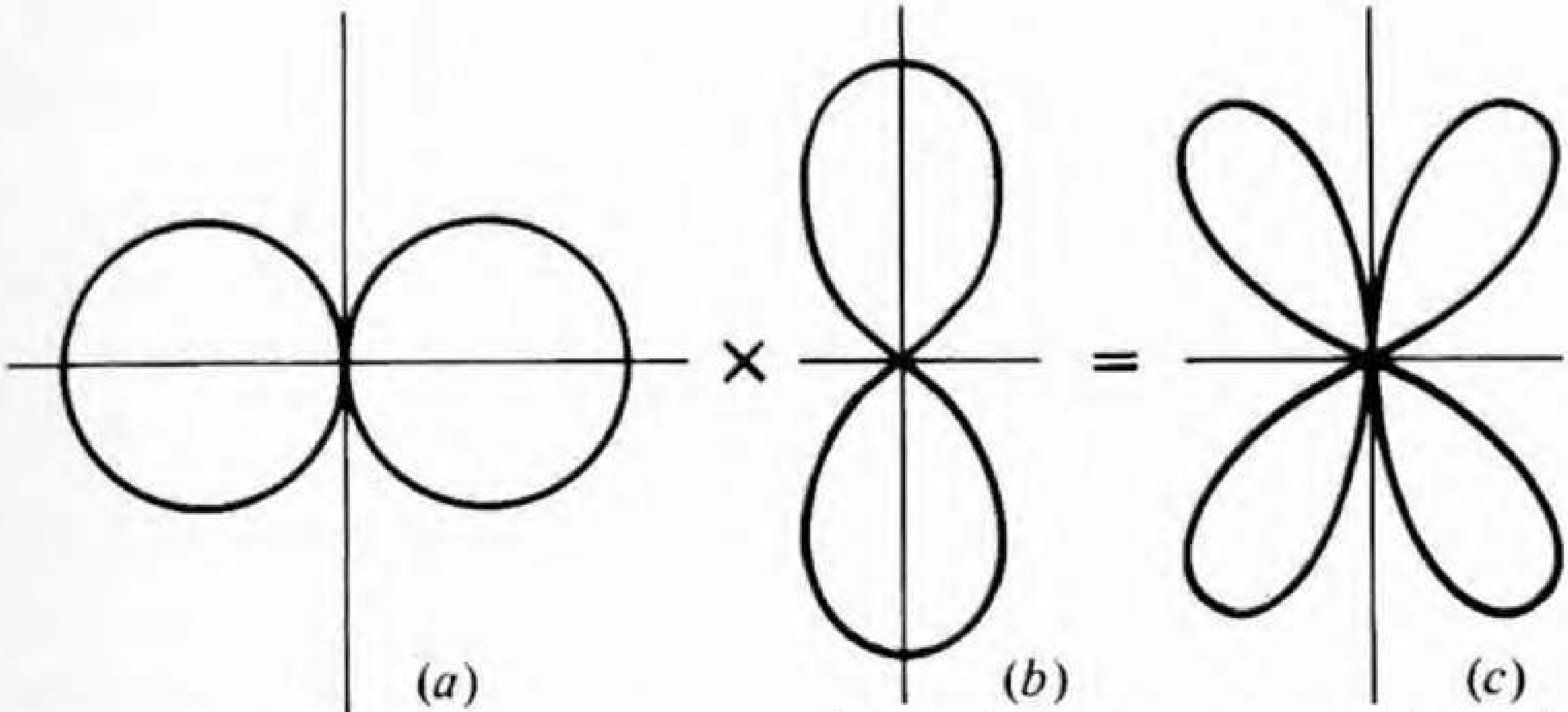
$$E = \cos \frac{\psi}{2}$$

$$\psi = d_r \cos \phi + \delta$$

$$d_r = \frac{2\pi d}{\lambda} = \beta d$$

Non isotropic point sources but similar Pattern Multiplication


×
=
sin φ
(a)
cos (π/2 cos φ)
(b)
E = sin φ cos (π/2 cos φ)
(c)



$\cos\phi$ $\quad$ $\cos\left(\frac{\pi}{2}\cos\phi\right)$ $\quad$ $E = \cos\phi\cos\left(\frac{\pi}{2}\cos\phi\right)$

2- Array of N-isotropic point Sources

Why antenna Array

1-Usually gain of single element is low, thus array is used for **increasing gain** for long distance communication

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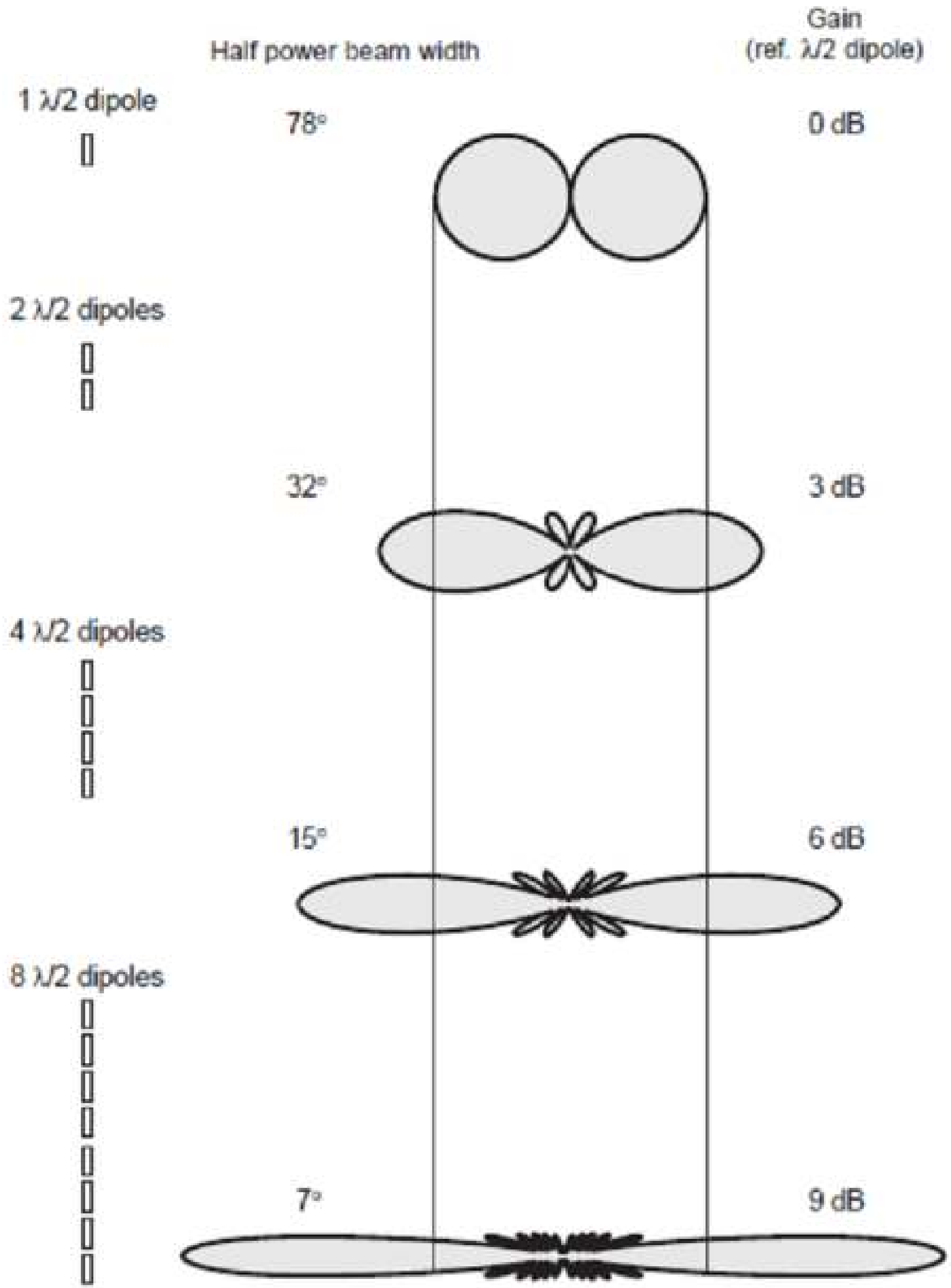
If λ/2 dipole is reference
i.e. its Gain considered to be=0dB
("note that λ/2 dipole has D=2dB")
then

2 element array increase gain by 3dB(double gain 2 time)

4 element array increase gain by 6dB(double gain 4 time)

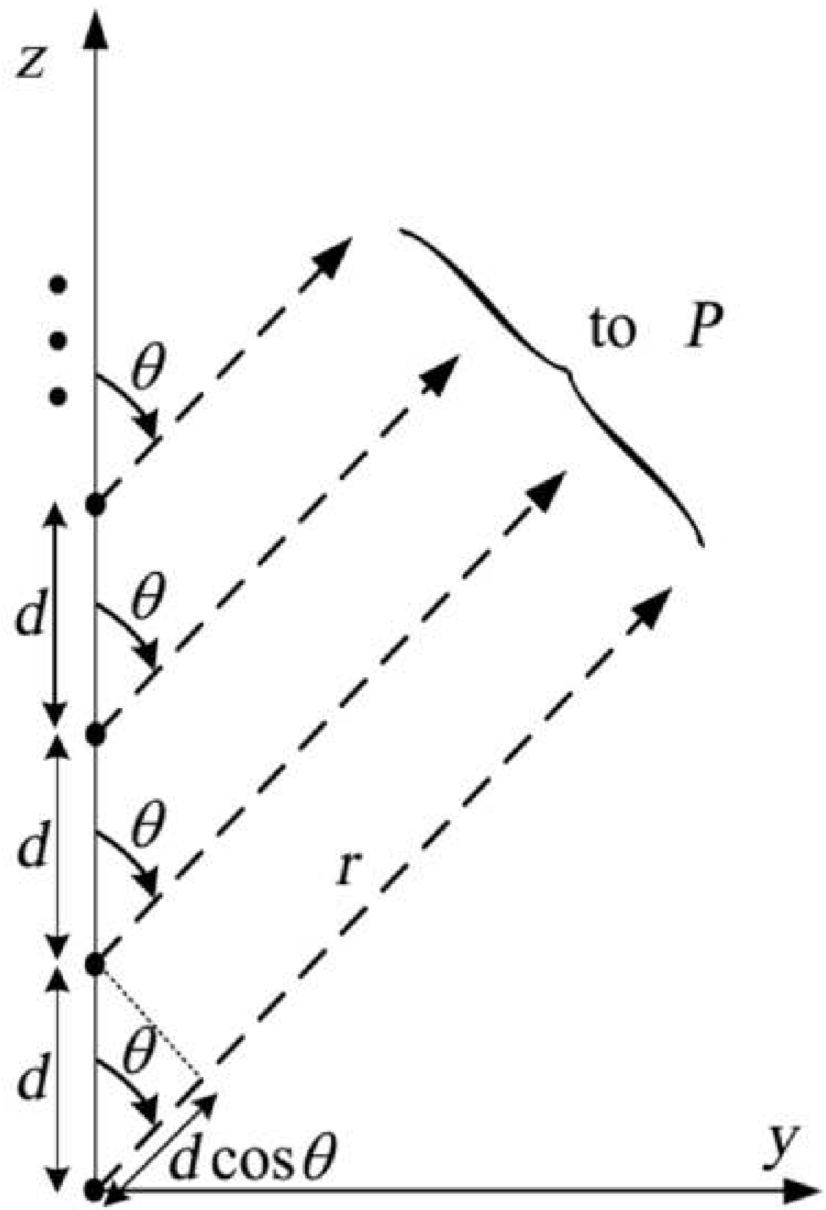
8 element array could increase gain by 9dB(double gain 8time)

2-Beam steering
by changing progressive phase
3-Nulling interference directions



Broad side array β=0

An array is said to be linear if the individual elements of the array are spaced equally along a line and uniform if the same are fed with currents of equal amplitude and having a uniform phase shift along the line



لاحظ الاثبات بفرضيتين

① الفرضية الأولى إن sources موضوعين على محور Z

② الفرضية الثانية الـ sources isotropic

The total resultant field at the distant point P is obtained by adding the fields due to n individual sources vectorically. Hence we can write,

$$E_T = E_0 \cdot e^{j0} + E_0 e^{j\psi} + E_0 e^{2j\psi} + \ldots + E_0 e^{j(n-1)\psi}$$

$$\therefore \quad \boxed{E_T = E_0 [1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}]} \quad \ldots (1)$$

$$\psi = kd\cos\theta + \beta.$$

- **$AF = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots\ldots + e^{j(N-1)\psi}$** *(1)*
- **$AF.\, e^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots\ldots + e^{j(N-1)\psi} + e^{jN\psi}$** *(2)*
- **Subtract (1) from (2)** **$AF(e^{j\psi} - 1) = (-1 + e^{jN\psi})$**

$$AF(e^{j\psi} - 1) = -1 + e^{+jN\psi}.$$

$$\begin{aligned} AF &= \left[\frac{e^{jN\psi} - 1}{e^{j\psi} - 1}\right], \\ &= e^{j[(N-1)/2]\psi}\left[\frac{e^{+j(N/2)\psi} - e^{-j(N/2)\psi}}{e^{+j(1/2)\psi} - e^{-j(1/2)\psi}}\right], \\ &= e^{j[(N-1)/2]\psi}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right] \end{aligned}$$

If the reference point is the physical center of the array

$$AF = \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right].$$

As the maximum value is N, when normalized,

$$AF_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right].$$

and

$$AF_n \simeq \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right].$$

(مقام=صفر)

to get $\underline{\underline{max}}$ put $\psi=0$ & use Lopital's Theorem

عشان لو وضعنا ψ=صفر هيدى البسط=صفر/المقام=صفر .. نستخدم لوبيتال ده

$\frac{\text{افاضل البسط}}{\text{افاضل المقام}}$ | وأعوض عن ψ=صفر (التفاضل بالنسبة لـ ψ)

$$|E_t| = E_o \frac{\sin(\frac{N\psi}{2})}{\sin(\psi/2)}$$

, after derivative

$$|E_{t_{max}}| = E_o \frac{(\frac{N}{2})\cos(\frac{N\psi}{2})\big|_{\psi=0}}{(\frac{1}{2})\cos(\psi/2)\big|_{\psi=0}}$$

$$\therefore |E_{t_{max}}| = N E_o \frac{\cos 0}{\cos 0} = N E_o$$

$$E_n = \frac{|E_t|}{|E_{max}|} = \frac{E_o \frac{\sin(\frac{N\psi}{2})}{\sin\psi/2}}{N E_o} = \frac{1}{N}\frac{\sin(\frac{N\psi}{2})}{\sin\psi/2}$$

Nulls , Maxima and Half power points

7) for nulls البسط = صفر مع استبعاد اصفار المقام

$$\sin\left(\frac{N\psi}{2}\right) = 0 \qquad \therefore \frac{N\psi}{2} = \pm n\pi$$

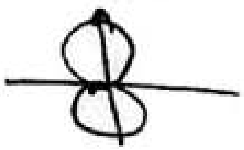
$$\therefore \psi = \frac{\pm 2n\pi}{N}$$

, here $n = 1, 2, 3, \ldots$

$n \neq 0$

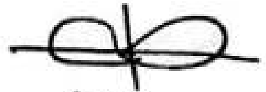
2) For Maxima   المقام = صفر أو … = صفر

$\theta = 0$



If $\psi = 0$ → endfire (Max at $\theta = 0, 180$)

→ Broadside (Max at $\theta = \pm 90$)



→ $\beta = 0$ دائماً

or we may use $\sin(\frac{\psi}{2}) = 0$ & $\frac{\psi}{2} = \pm m\pi$

$m = 0, 1, 2$

→ main lobe

3) FNBW .. first null Beam width $= 2|\theta_{max} - \theta_{m}|$

→ من مكانها في خطوة 2

→ قيمة الزاوية عند أول تعويض في حالة ال null ($m = 1$)

4 half power points $E = \pm \frac{1}{\sqrt{2}}$ at $\frac{N\psi}{2} \cong \pm 1.391$

$\psi = kd\cos\theta_h + \beta$

5 $HPBW = 2|\theta_{max} - \theta_h|$

6 minor (side Lobes) Maximax at $\sin\left(\frac{N\psi}{2}\right) = \pm 1$

$\therefore \frac{N\psi}{2} = \pm (2S+1)\pi/2$

7 First side lobe Beamwidth $= 2|\theta_{max} - \theta_s| \rightarrow$ at $S=1$

Zatoona for Nulls , Maxima and Hp

It is required to study $(AF)_n$

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

- **Nulls**

$$N\frac{\psi}{2} = \pm n\pi \quad , \quad m = 1,2,3,.. \neq 0, N,2N,....$$

- **Maximum**

$$\frac{\psi}{2} = \pm m\pi \quad , \quad m = 0,1,2,.... \; (\text{ 0 for main lobe })$$

Grating lobe condition (at m=1,2,3,...)

- **3-dB point**

$$N\frac{\psi}{2} = 1.39$$

- **Secondary Maximum for minor lobes**

$$N\frac{\psi}{2} = \frac{2s+1}{2}\pi \quad , \quad s = 1, 2, 3,.....$$

Maximum of first minor lobe occurred at $N\psi/2=3\pi/2$

End fire – Broad side

Broadside Arrays

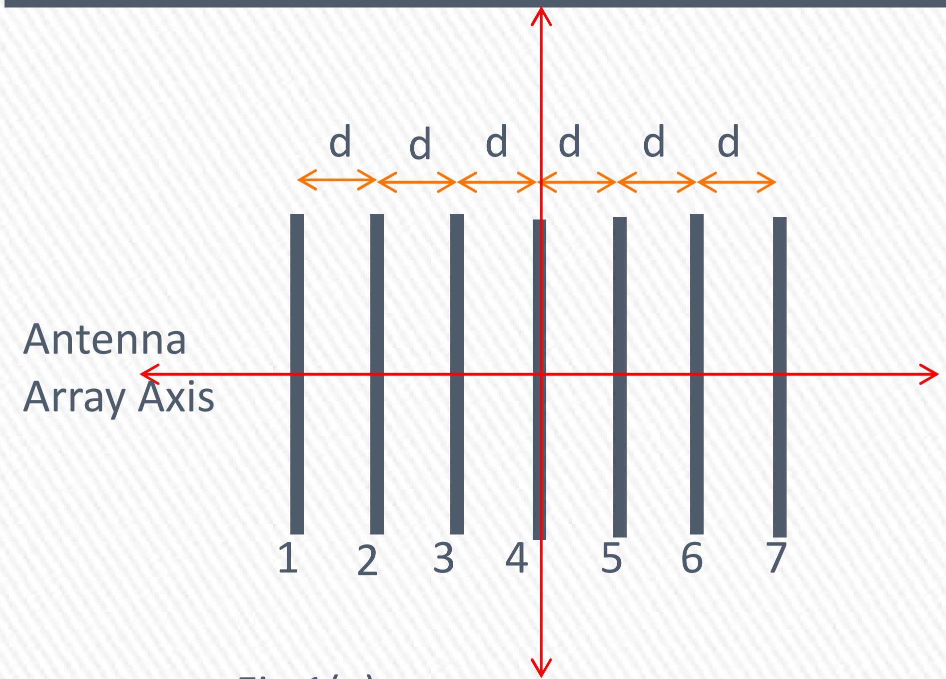


Fig 1(a)

[Link]

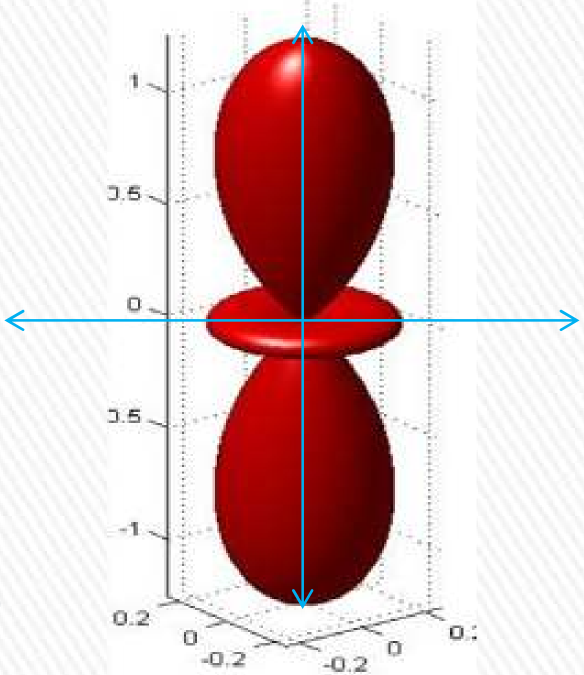


Fig 1(b)

- The radiation pattern of broadside array is perpendicular to the line of array axis and bidirectional.

- The broadside array is bidirectional which radiates equally in both direction of maximum radiation.

- The broadside array may be defined as "it is an arrangement in which the principle direction is perpendicular to the array axis and also the plane containing the array element.

End-fire Arrays

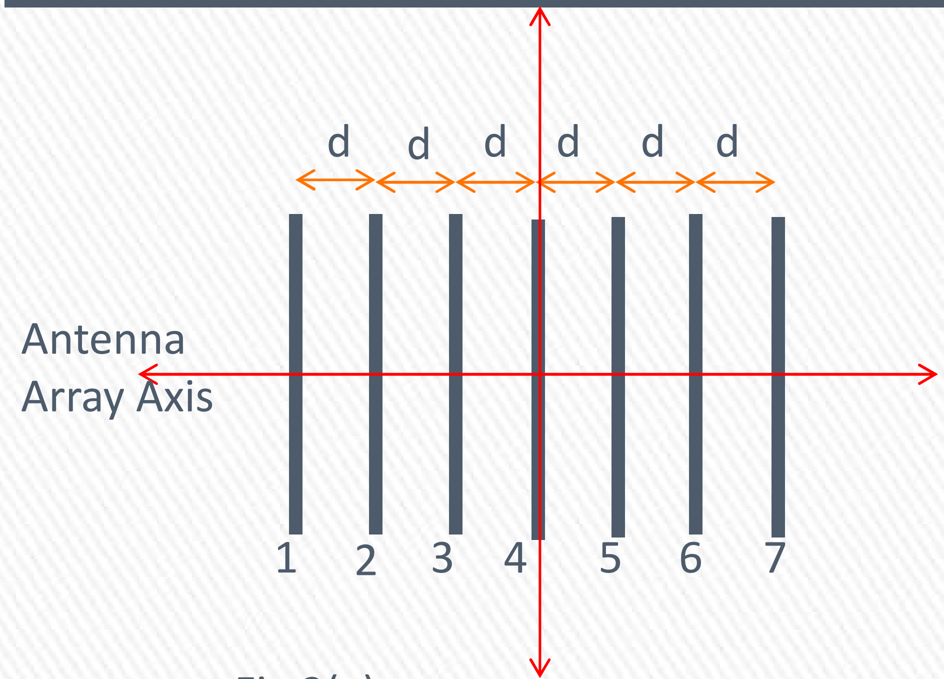


Fig 2(a)

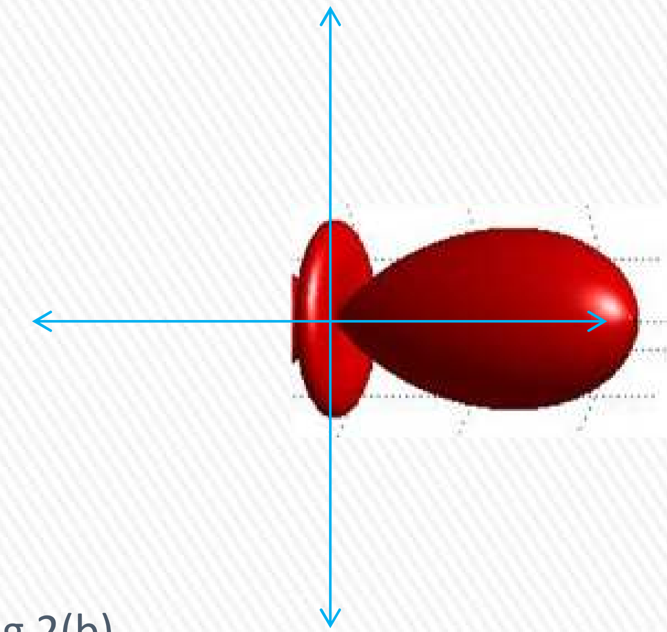
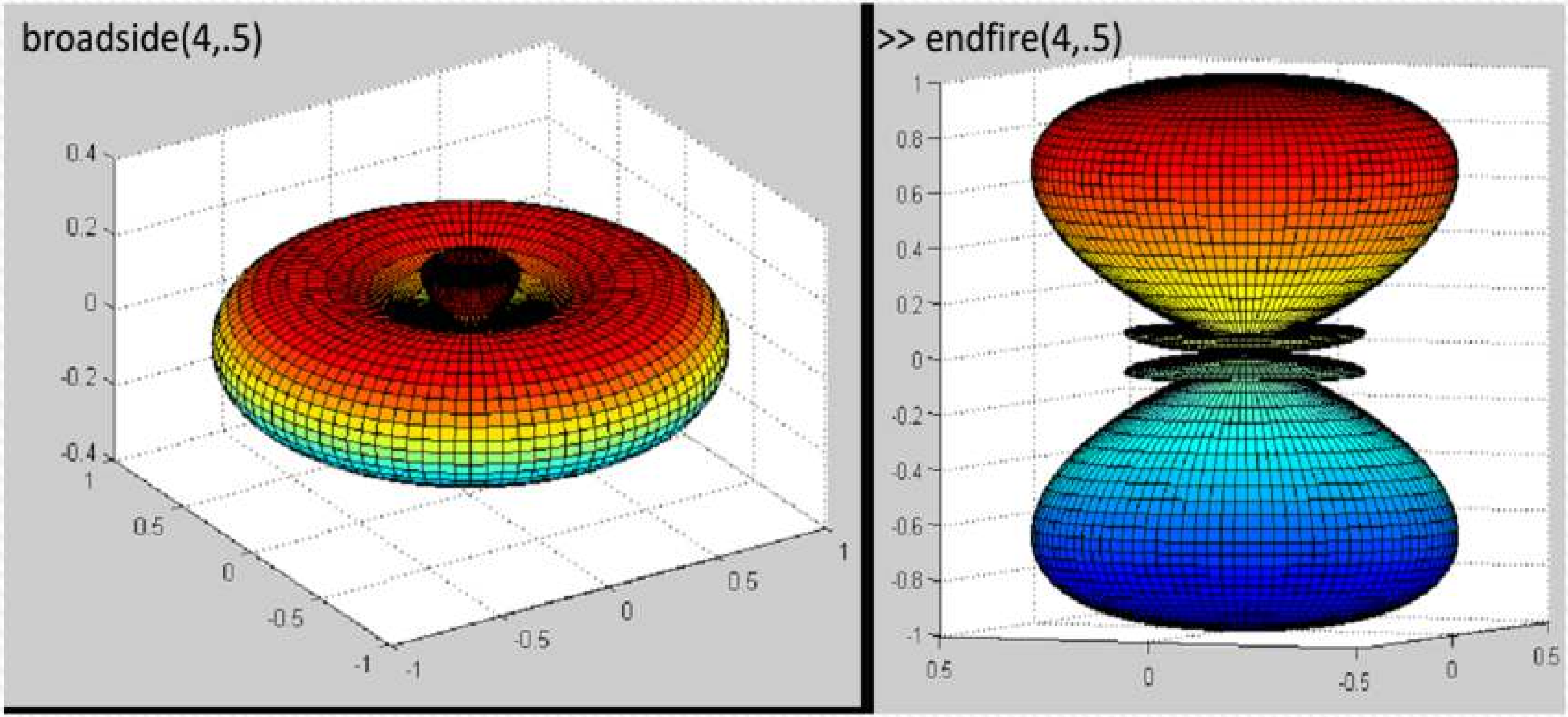


Fig 2(b)

- Instead of having the maximum radiation normal to the axis of the arrays it may be desirable to direct it along the axis of the array.

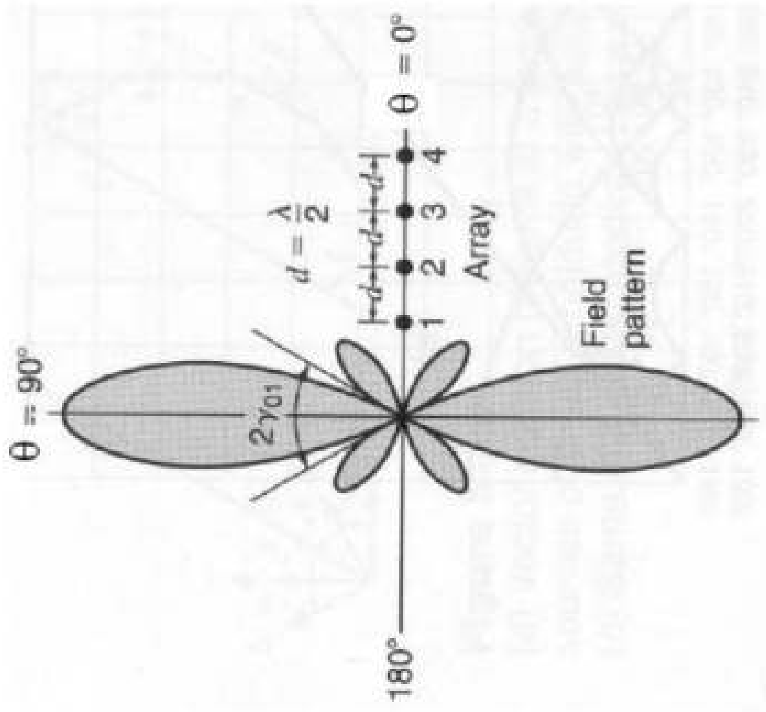
- However, the end fire arrays is same as the broadside array but the **individual element are fed in out of phase(i.e. 0° or 180°)**



broadside(4,.5)
0.4
0.2
0
-0.2
-0.4
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
>> endfire(4,.5)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0.5
0
-0.5
0
0.5

- Max occurred at $\psi = 0 = k.d.\cos\theta + \beta$ (for AF pattern)

Broad Side Array

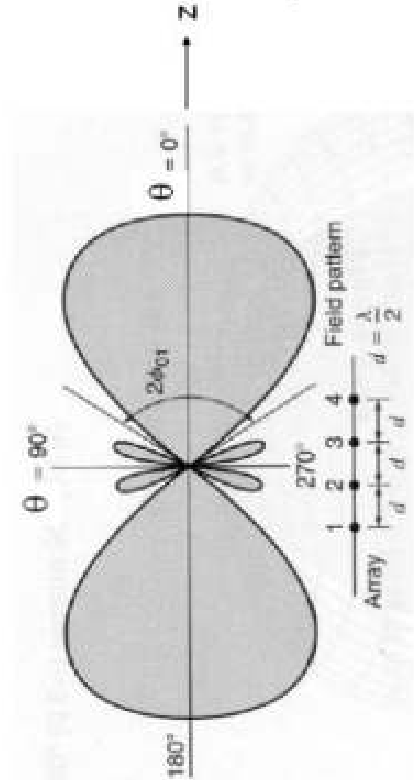


Since it is desired to have the first maximum directed toward $\theta_0 = 90°$, then

Setting For broad side AF pattern

$$\psi = kd\cos\theta + \beta|_{\theta=90°} = \beta = 0$$

End Fire Array



Setting For End Fire AF pattern

To direct the first maximum toward $\theta_0 = 0°$,

$$\psi = kd\cos\theta + \beta|_{\theta=0°} = kd + \beta = 0 \Rightarrow \beta = -kd$$

If the first maximum is desired toward $\theta_0 = 180°$, then

$$\psi = kd\cos\theta + \beta|_{\theta=180°} = -kd + \beta = 0 \Rightarrow \beta = kd$$

$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

- NULLS

- Nulls occurred at $\sin(N\psi/2)=0$
- Kdcosθ+ β=±2nπ/N   where n=1,2,3 (again n≠ 0 or N or 2N.......this make $(AF)_n$=0/0 which is max condition)

$$\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$$

- **Broadside Array (sources in phase β=0)**

$$\theta_n = \cos^{-1}\left(\pm\frac{n}{N}\frac{\lambda}{d}\right)$$
$$n = 1, 2, 3, \ldots$$
$$n \neq N, 2N, 3N, \ldots$$

End fire Array (β=-kd)

$$\theta_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right)$$
$$n = 1, 2, 3, \ldots$$
$$n \neq N, 2N, 3N, \ldots$$

1- because $\cos^{-1}$(less than 1)

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right] \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]$$

- MAXIMUM
- Maximum occurred at $\psi/2 = \pm m\pi$ (for $(AF)_n = 0/0$)
- Kdcosθ+ β=±2mπ where m=0,1,2,3

$$\theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right]$$

$$(\mathrm{AF})_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right] \simeq \left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi}\right]$$

- 3-dB point for AF
- Use Approximation sin(x)/x because it does not depend on N

Using try and error 3dB occurred at sin(x)/x=.707 i.e. x=1.39

*because it is field pattern (sin(1.93*180/π)/1.39=.7076*

x	$\sin(x)/x$
1.3	0.74120
1.4	0.70389

$$\frac{N}{2}\psi = \frac{N}{2}(kd\cos\theta + \beta)|_{\theta=\theta_h} = \pm 1.391$$

$$\Rightarrow \theta_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]$$

- **Broadside Array (sources in phase β=0)**

HALF-POWER POINTS $\quad \theta_h \simeq \cos^{-1}\left(\pm\frac{1.391\lambda}{\pi N d}\right)$

End fire Array (β=-kd)

$$\theta_h \simeq \cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right)$$

Approximate Directivity for End fire and Broad Side

For end fire

$$\text{Directivity} = \frac{41253}{\theta_{HP}^{\circ}\,\phi_{HP}} \quad \text{or} \quad \approx 4N\left(\frac{d}{\lambda}\right)$$

N → no. of sources

only in case $L \gg d$

array length & $L = (N-1)d$

Broad side

$$D = \frac{41253}{\theta_{HP}\,\phi_{HP}}$$

or $\approx 2N\left(\frac{d}{\lambda}\right)$

Only in case $L \gg d$

or $L = (N-1)d$

Zatoona for End fire & Broad Side

For array of N Isotropic Point sources:

$$(AF)_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}\right]$$

Array/parameters	Ordinary End-fire $(\theta_o=0^o)$	Broad-side
	$\psi = kd\cos\theta + \beta\vert_{\theta=0^\circ} = kd + \beta = 0 \Rightarrow \beta = -kd$ $\theta_0 = 180^\circ$, $\psi = kd\cos\theta + \beta\vert_{\theta=180^\circ} = -kd + \beta = 0 \Rightarrow \beta = kd$	$\psi = kd\cos\theta + \beta\vert_{\theta=90^\circ} = \beta = 0$ $\theta_0 = 90^\circ$.
Nulls (sin (nΨ/2)=0) n≠0, n=1, 2, ...	$\theta_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right)$	$\theta_n = \cos^{-1}\left(\pm\frac{n}{N}\frac{\lambda}{d}\right)$
Maxima (sin (Ψ/2)=0) N=0, 1, ...	$\theta_m = \cos^{-1}\left(1 - \frac{m\lambda}{d}\right)$	$\theta_m = \cos^{-1}\left(\pm\frac{m\lambda}{d}\right)$
First Null Beam-width =2│θmax-θ1n│... at n=1	$\Theta_n = 2\cos^{-1}\left(1 - \frac{\lambda}{Nd}\right)$	$\Theta_n = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{Nd}\right)\right]$
Half power Points nΨ/2 = ±1.391	$\theta_h \simeq \cos^{-1}\left(1 - \frac{1.391\lambda}{\pi dN}\right)$	$\theta_h \simeq \cos^{-1}\left(\pm\frac{1.391\lambda}{\pi Nd}\right)$
Half Power Beam-width =2│θmax- θh│	$\Theta_h \simeq 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi dN}\right)$	$\Theta_h \simeq 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.391\lambda}{\pi Nd}\right)\right]$
Directivity (if L>>d) Where L = (N-1)d	4*N*(d/λ)	2*N*(d/λ)

Examples

1. Design a four –element ordinary end fire array with the elements placed along the Z-axis a distance d apart with the maximum of the array factor directed toward $\theta=0^{o}$. for a spacing of $d=\lambda/2$ between the elements find the

   (a) Progressive phase excitation between the elements to accomplish this.
   (b) Angles (in degrees) where the nulls of the array factor occur.
   (c) Angles (in degrees) where the maximum of the array factor occur.
   (d) Beam width (in degrees) between the first nulls of the array factor.
   (e) Directivity (in dB) of an array factor.

Sol.

a- $\psi = kd\cos\theta + \beta$; $\psi = 0$ at $\theta = 0^\circ$ (given)

$\therefore 0 = kd + \beta$ or $\beta = -kd = -\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = -\pi = -180^\circ$

b- nulls. $\sin\left(\frac{N\psi}{2}\right) = 0$ $\therefore \frac{N\psi}{2} = \pm n\pi$; $n = 1, 2, \dots$ (4)

$\therefore \frac{4}{2}(kd\cos\theta + \beta) = \pm n\pi$

$2(\pi\cos\theta - \pi) = \pm n\pi$

$\therefore \cos\theta - 1 = \pm\frac{n}{2}$ or $\boxed{\cos\theta = 1 \pm \frac{n}{2}}$

at $n=1$ $\cos\theta = 1 \pm \frac{1}{2}$ → $\frac{3}{2}$ X, → $\frac{1}{2}$ $\therefore \boxed{\theta = \pm 60^\circ}$

at $n=2$ $\cos\theta = 1 \pm \frac{2}{2}$ → 2 X, → 0 $\boxed{\pm 90^\circ}$

at $n=3$ $\cos\theta = 1 \pm \frac{3}{2}$ → $\frac{5}{2}$ X, → $-\frac{1}{2}$ → $\boxed{\pm 120}$

at $n=4$ $\cos\theta = 1 \pm \frac{4}{2}$ → 3 X, → -1 X

null at 180 [illegible]

c- angles of max

$\sin\left(\frac{\psi}{2}\right) = 0$ or $(\pi\cos\theta - \pi) = \pm m\pi$

$\therefore \cos\theta = (1 \pm m)$

for $m = 0$ $\therefore \cos\theta = 1 \longrightarrow \theta = 0^\circ$

for $m = 1$ $\cos\theta = 1 \pm 1 \to 2,\ 0 \to \theta = 90$ X null دي صفرية

for $m = 2$ $\cos\theta = 1 \pm 2 \to 3$ X, $\to (-1)$ $\theta = 180^\circ$ ✓

for $m = 4$ Refused

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d- $FNBW = 2|\theta_{max} - \theta_{min}| = 2|0 - 60| = 120^\circ$

θ_{min} → اول زاوية صفرية

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e- $D = 4N\left(\frac{d}{\lambda}\right) \longrightarrow$ because $L = (N-1)d$

$L = 3d$

$L > d$

$= 4 \times 4 \times \frac{\lambda/2}{\lambda} = 8$

حلة آخر باحتساب θ_{Hp} ، $D = \frac{41253}{(\theta_{Hp})^2}$

$$\theta_{Hp} = 2\,|\theta_{max}^{0} - \theta_n|$$

$$\theta_n \rightarrow \frac{\psi}{2} = \pm 1.391$$

$$2(\pi \cos\theta - \pi) = \pm 1.391$$

$$2 \times 3.14\,[\cos\theta - 1] = \pm 1.391$$

$$\cos\theta_n = 1 \pm \frac{1.391}{2 \times 3.14}$$

فيه + مرفوضة

$$1 - \frac{1.391}{2 \times 3.14}$$ ← نطلع منها الزاوية

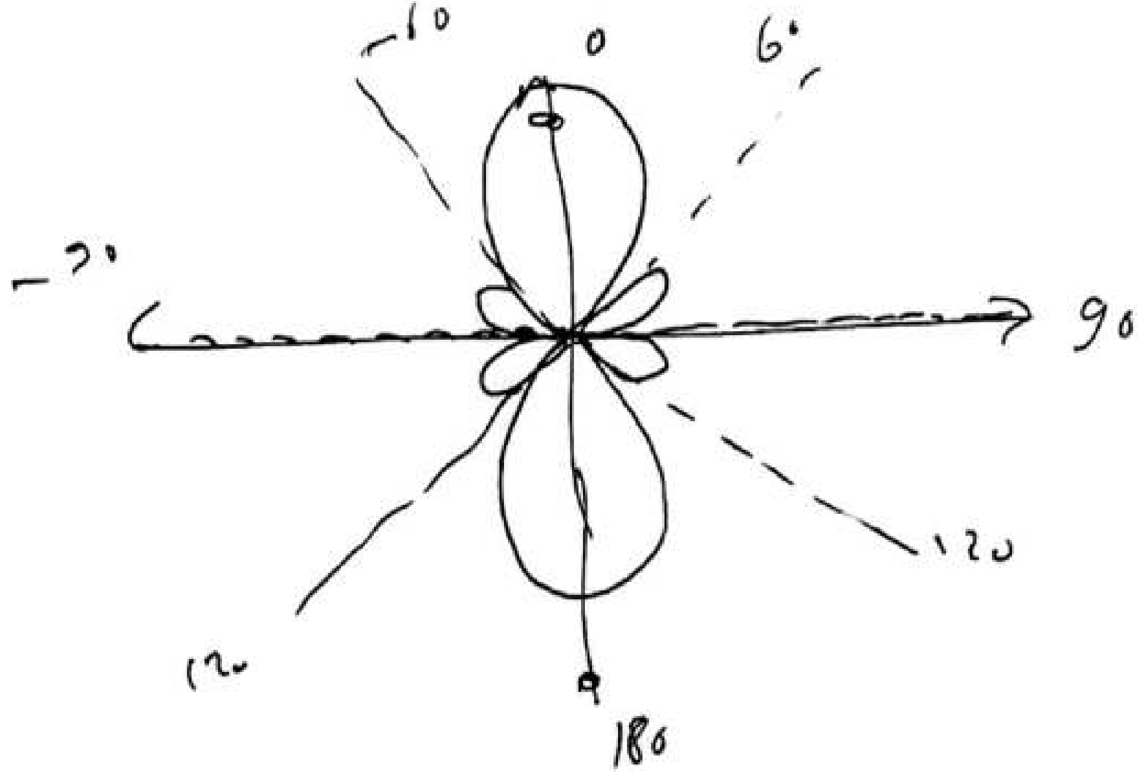
ونكمل الحل

endfire

null

±60 , ±90 , ±120°

max 0, 18



Example (2)

Arrays of 10 isotropic elements are placed along z-axis a distance $d=\lambda/4$ apart. Assuming uniform distribution. Find for both broadside and ordinary end-fire cases the following:

(a) Progressive phase (in degrees).

(b) First side lobe level beam width.

(c) Directivity (in dB).

Sol. [a] For Broadside (max at $\theta = \pm 90$, $\beta = 0$)

$\therefore \psi = kd\cos\theta + 0 = kd\cos\theta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\cos\theta = \frac{\pi}{2}\cos\theta$

→ $\beta = 0$

→ $D = 2N\left(\frac{d}{\lambda}\right) = 2 \times 10\left(\frac{\lambda/4}{\lambda}\right) = 5 \approx 6.9\,dB$

↳ $L = (N-1)d = 9d$

→ $FSLBW = 2|\theta_{max} - \theta_{1s}|$

↳ 90

$\theta_{1s} \rightarrow \sin\left(\frac{N\psi}{2}\right) = \pm 1$ or $\frac{N\psi}{2} = \pm\left(\frac{2S+1}{2}\right)\pi$

at $S = 1 \rightarrow \theta_{1s}$ $\therefore \frac{10}{2}\left[\frac{\pi}{2}\cos\theta\right] = \pm 3\pi/2$

$\therefore 5\cos\theta = \pm 3$ or $\cos\theta_{1s} = \pm\frac{3}{5}$ → $\frac{3}{5}$ 53°, → $-\frac{3}{5}$ 127°

$FSLBW = |127 - 53| = 74°$ or $2|90 - 53| = 74°$ ★

$\%$ لوطلب

HPBW $\rightarrow$ $\frac{N\psi}{2} = \pm 1.391$

$5\left[\frac{\pi}{2}\cos\theta_h\right] = \pm 1.391$

$\therefore \cos\theta_h = \pm \frac{1.391 \times 2}{5\pi}$

$\theta_h = 79.8 , 100.2$

$\theta_{Hp} = 100.2 - 79.2 = 20.4^\circ$

or $= 2|\theta_{max} - \theta_h| = 2|90 - 79.2| = 20.4^\circ$

$\%$ لوطلب

FNBW $= 2|\theta_{max} - \theta_{1n}|$

($\theta_{max} \rightarrow 90$)

$\theta_{1n} \rightarrow \sin\left(\frac{N\psi}{2}\right) = 0 \quad m=1$

$\frac{N\psi}{2} = \pm m\pi$

$5\left(\frac{\pi}{2}\cos\theta\right) = \pm \pi$

$\cos\theta = \pm \frac{2}{5} \rightarrow 66.4^\circ, \ 113.6$

$\therefore$ FNBW $= 113.6 - 66.4 = 47.2^\circ$

or $= 2|90 - 66.4| = 47.2^\circ$

4 - Array of N (non-isotropic sources) (lie on Z)

If sources are Non-isotropic Ex (infinitesimal)

$\therefore \quad \bar{E}_{n_{total}} = \bar{E}_{n_{dipole}} \times AF_{\text{of isotropic}}$

$$= (\sin\theta) \frac{\sin\left(\frac{N\psi}{2}\right)}{N \sin \psi/2}$$

If sources are of $\frac{\lambda}{2}$ antenna

$$\therefore \quad E_{total} = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \times \frac{1}{N} \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin \psi/2}$$

Next Lecture (7)

Chapter(4): Wire Antennas
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Thank You